Sieve of Eratosthenes

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Grade Levels: 5-12

Creator Skill Level: 1

Subject Area: Mathematics

World: Prime Sieve TG.world

THE PROJECT

Prime numbers are numbers that can be divided only by themselves or by 1. Prime numbers are important for doing mathematical operations such as adding or reducing fractions. A classic technique for finding Prime numbers is to use the Sieve of Eratosthenes.

Prime Sieve TG.world allows students to find primes using the Sieve of Eratosthenes, one step at a time. Traditionally, the Sieve is done on a sheet of paper and the process becomes mechanical instead of focusing on why numbers are dropped out of the Sieve. Using the Prime Sieve world, the focus is on divisors.

This makes an excellent hands-on and visualization aid when working with arithmetic concepts such as reducing fractions, adding and subtracting fractions, or finding the Least Common Multiple or Greatest Common Factor of numbers. Any skill that requires the students to factor numbers would benefit from this exercise.

LEARNING OBJECTIVES

NCTM Standards 2000: Mathematics instructional programs should foster the development of number and operations sense so that all students:

- Understand numbers, ways of representing numbers, relationships among numbers and number systems
- Understand the meaning of operations and how they relate to each other
- Use computational tools and strategies fluently and estimate appropriately

The purpose of this activity is to gain a better understanding of prime numbers using the Sieve of Eratosthenes.

SUGGESTED FORMAT

The teacher should introduce or review the definition of prime numbers. Then if possible the teacher should demonstrate Prime Sieve TG.world for the entire class. Once students get the idea of how the Prime Sieve world works, they can explore further in small groups at the computer. The focus should be on understanding why many numbers drop out of the sieve and why only prime numbers are left.

STUDENT TASKS

Students will be given time to explore Prime Sieve TG.world and ask questions about how the numbers are removed from the list. Working in small groups they will activate the world and observe each pass of the arrow across the list of numbers. Challenge them to make sure they understand why certain numbers are being removed from the list. Why did numbers such as 8, 10, and 12 get removed at the same pass? What is the step used to determine the numbers that will be removed on the second pass of the arrow, the third pass, and so on?

Students should also be challenged to consider the limitations they face with the sieve if they are trying to determine if a very large number is actually a prime number.

TIPS

The Sieve of Eratothenes is an ancient method for finding primes. It is a purely mechanical method that begins with a grid of numbers. First, every multiple of 2 is crossed out. Then every multiple of 3 is crossed out. Since 4 is already crossed out, the next number selected is 5. Then each multiple of 5 is crossed out. This process can be continued indefinitely.

Older students could be asked to figure out at what point they no longer step through the process of crossing out numbers. For example, if you have a grid of numbers from 1 to 100, then you can stop checking for primes at 11 because 11 x 11 is greater than 100. If you have a grid of 1 to 1000, you can stop at 37 because 37 x 37 is greater than 1000.

Here are the answers to questions at the end of the student worksheet:

- The Sieve world is very mechanical. When people do the Sieve "by hand," they generally count to the next number and don't cross out numbers already crossed out in the grid. The actual Sieve algorithm says "Move to the next uncrossed-out number and begin crossing out multiples of this number." In fact, the Sieve world simply steps to the next number. Use this as a discussion as to how computers are capable of doing mechanical computations mindlessly but that human ingenuity allows us to make more efficient decisions.

- You can be sure all the numbers left are primes because all divisors have been tested.

- To determine if 357 is a prime number, you would first extend the Sieve up to 357 and then go through the complete process of crossing out divisors.

- Start by checking the prime numbers that have been found by the Sieve. Is 112 divisible by 2? Is 345 divisible by 2? Then ask if 112 is divisible by 3 and if 345 is divisible by 3. Continue until you find a prime number that divides both numbers without remainders.

- The physical size of the Sieve and the amount of time used would probably be your limiting factors.

Locate an article about prime numbers found by a computer, such as Chris Caldwell's *The Largest Known Prime by Year: A Brief History* located at **www.utm.edu/research/primes/notes/by_year.html.** You might also have students check *The Largest Known Primes,* found on the Web at **www.utm.edu/research/primes/largest.html#intro**. Use these articles when you discuss this activity with your class. Help them to understand that while the Sieve of Eratosthenes was one of the "old" ways to find prime numbers, technology has provided the means for letting computers take over the task. Chris Caldwell's comments are particularly helpful in providing an example of why the "older" methods have proven to be less useful than computers.

CHALLENGES

Review the places that primes are useful. Do some examples of reducing fractions and adding fractions to see how knowing prime numbers is helpful.

Read the "about this world" for Prime Sieve TG.world for additional ideas for changes.

DISCUSSION TOPICS

Why does the Sieve of Eratosthenes work? (It removes all numbers with divisors other than themselves in a systematic way.) Could the Sieve of Eratosthenes be infinitely large? (Yes, but it would then take an infinite amount of paper and take infinite time to complete.) How many prime numbers are there? (Infinite.) Do prime numbers ever stop? (No.)

TEACHER PREPARATION

If the teacher is going to use a computer to demonstrate the Prime Sieve world, then a projection system is needed. The teacher might also demonstrate using a grid on an overhead projector, and if so, an overhead needs to be prepared.

The Prime Sieve world must be loaded on every computer along with either Stagecast Creator or Stagecast Player.

ASSESSMENT SUGGESTIONS

When doing arithmetic practice, have students indicate the prime numbers that they used.

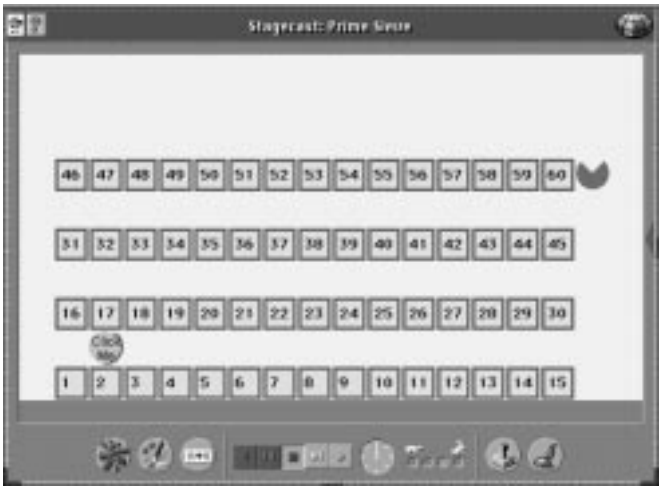
Have students create a list of tips that could be used to help them eliminate numbers that are not prime such as:

- Any even number is divisible by 2 and hence not prime.
- Any number that ends in a 5 is divisible by 5 and hence not prime.
- If the number created when adding the digits of any number is divisible by 3, the original number is divisible by 3.

STUDENT WORKSHEETS

We are going to explore the Sieve of Eratosthenes as a means of finding prime numbers. Eratosthenes lived from 275-194 B.C. in Greece. He was considered a university genius. He was director of the great library at Alexandria and distinguished himself in mathematics, poetry, philosophy, history, and geography. Erathosthenes' method for finding prime numbers remained the only way to find prime numbers for over 2000 years.

Open Prime Sieve TG.world. You see this screen:



Notice that there is a grid of numbers from 1 to 60. At the left of the screen is a button that says "Click me." When you click on the button, an arrow moves through the Sieve. For example, the first time you click on the arrow, it moves through the Sieve crossing off multiples of 2. Observe that numbers such as 2, 4, 6, 8, 10, and so forth are removed.

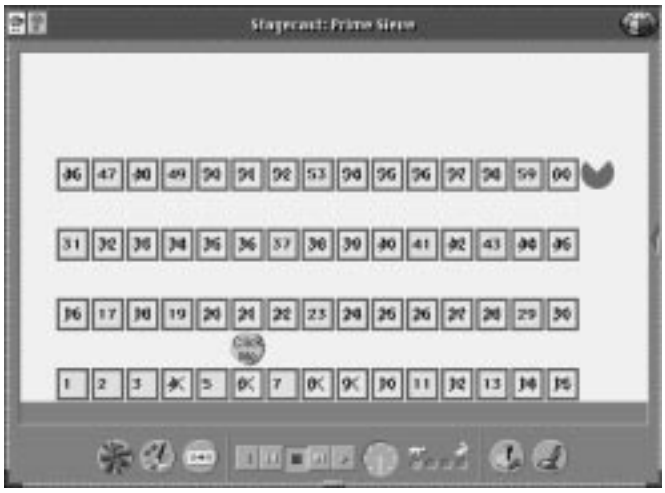
Click on the button several times so that it moves all the way across the first line of boxes. Examine the remaining numbers. What name is used for the type of numbers left in the display? What do you think might be the criteria for removing numbers on the next pass through the Sieve?

Next, select Reset from the Creator menu.

Then set the speed to slow.

Click on the button again. Watch as the Sieve crosses out multiples of 2. Click again. Watch carefully. What numbers are crossed out? Click a third time. Why does nothing happen when the button is above 4? Next the button moves to 5. Before you click on the button again, predict what the arrow will do. Try it. Were you correct?

At this point, how many prime numbers have you found? Does your screen look like this?



Continue to run the program. When you have run all of the numbers allowed by the program, what numbers are left? How can you be absolutely sure they are all primes?

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Stop the world and double-click on any one of the pink squares. Click on the tab to open the variables. You see a variable called factor. The value of "factor" tells you which number removed the number in the box from the list of primes.

Have you noticed that the arrow that crosses out numbers appears to cross out numbers that are already crossed out? If you were doing this activity "by hand" how would you handle this situation?

Here are further questions for you to consider:

Keep the Sieve patterns in mind and think about the steps you would use to determine if 357 is a prime number.

Keep the power of the Sieve in mind and consider the steps you would use to decide on the Lowest Common Denominator for two fractions if the first one has a denominator of 112 and the second has a denominator of 345.

Discuss the circumstances where you would no longer want to apply the Sieve rules to decide if a given number was prime. What factors influenced your decision?